NPV OPTIMIZATION

Multiple Project Alternatives NPV with Variable Costs

Suppose that there are several alternatives for project execution.

Each alternative includes set of tasks with intervals of their costs.
For each task, transaction date is defined: date for performing payments or receipts.

All payments/receipts are capitalized:

$S_{cap} = S(1+r)^d$

where

S is cost before capitalization;

S_{cap} is capitalized cost (NPV of the cost);

r is annual capitalization percent (say 5%);

d is time difference between date of start of the project and date of a transaction (say, -5 days)

Note 1: Because capitalization percent is defined for a year, time difference is measured in parts of year. So 5 days = 5/365.

Note 2: , time difference is supposed as negative, so $S_{cap} < S$.

Time relation exists between tasks due to technological cycle.
Also, alternatives definite relations exist between alternatives (>,=,<).

The goal is to maximize total NPV of the entire project for one of the alternatives by changing dates and costs of transactions subject to their constraints (costs intervals and time relations) and to the general constraint:

$NPV_1 \leq NPV_2 \leq ... \leq NPV_k$.

Objective function is:

$$NPV_1 = \sum_i S_i^1 (1+r_1)^{-d_i^1} \xrightarrow{d} Max,$$

Where

S_i^k are control variables: negative payments or positive receipts for task i in project alternative k;

S_i^k is subject to constraints

$m_i^k \leq S_i^k \leq M_i^k$

where

m_i^k, M_i^k are constants,

$d_i^k > 0$ are control variables: after how many days from initial date begins task i in project alternative k;

$r_k \approx 0.05$ is bank interest in project alternative k;

$d_i^k > 0$ are subject to constraints

$0 \leq d_k^i \leq F_k$ (project alternative k time frame);

$d_i^k - d_j^k \leq D_{ij}^k$ (time relation between task i and task j in project alternative k).

Between optimal values of all project alternatives there are general relations:

$\mathrm{NPV}^1 \leq NPV^2 \leq ... \leq NPV^K$.

$k = 1,2,...,K$ is current number of the project alternative;

K is total number of the project alternative;

Without these relations costs of tasks S_i^k would be just m_i^k or M_i^k depending on task supposed for payment or receipt.

Complex Tasks in NPV

Minimal and maximal stages numbers must be entered for each task. For a simple task both are the same (and equal = 1). But for so called *complex* tasks these numbers will be limits for the optimal value searching.

Lets denote minimal and maximal stages numbers of the task i as follows: $S^i_{\min}, S^i_{\max}$. Then for complex task # i with cost C_i that includes V_i stages NPV will be:

$$NPV^i = \sum_{j=1}^{V_i} \frac{C_i}{V_i (1+r_i)^{d_i + j \cdot D_i}},$$

where

$S^i_{\min} \le V_i \le S^i_{\max}$,

D_i is duration of each stage of the task i,

r_i is banking annual rate for cost of task i,

d_i is the date of the task i.

After separating constant and variable values, NPV will be:

$$NPV^i = \frac{C_i}{V_i (1+r_i)^{d_i + D_i}} \sum_{j=0}^{V_i - 1} (1+r_i)^{-jD_i}$$